SOLUTION SET

Chapter 13

SPECIAL LASER CAVITIES AND CAVITY EFFECTS

"LASER FUNDAMENTALS"

Second Edition

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1. In designing an unstable resonator for a 248-nm KrF excimer laser, for practical considerations it is desirable to have a relatively long gain medium such that the separation between mirrors is 0.5 m. For a magnification factor of 5, what would be the radii of curvature of both the output mirror and the rear mirror? If the laser cavity were to be designed for minimum diffraction losses, what would be the diameter of the output mirror?

\[ M = \frac{R_r}{R_o} = 5 \]

\[ R_r - R_o = 2d \]

\[ 5R_o - R_0 = 2d \]

\[ R_0 = \frac{2d}{4} = \frac{d}{2} = 0.25 \text{m} \]

\[ R_r = 1 \text{m} + 0.25 \text{m} = 1.25 \text{m} \]

\[ N_{eq} = \left[ \frac{(M-1)}{2M^2} \right] \left( \frac{a^2}{\lambda d} \right) = \left( \frac{5-1}{2 \cdot 5^2} \right) \frac{a^2}{\lambda d} = 0.5 \]

\[ a^2 = (0.5) \lambda d \left( \frac{50}{4} \right) = (0.5)(248 \times 10^{-9})(0.5) \frac{50}{4} \]

\[ a = 8.8 \times 10^{-4} = 0.88 \text{mm} \]

\[ \text{diameter} = 2 \times 0.88 = 1.76 \text{mm} \]
2. Beginning with (13.4), the constraint for a positive-branch confocal resonator that $R_r - R_o = 2d$, obtain (13.10) and (13.11).

$$R_r - R_o = 2d$$

but $R_o$ has negative curvature so we replace $R_o$ with $-R_o$ to obtain

$$R_r + R_o = 2d \text{ or } \frac{R_r}{2} + \frac{R_o}{2} = d$$

$$q_1 = 1 - \frac{d}{R_1} \quad q_2 = 1 - \frac{d}{R_2}$$

$$q_1 = 1 - \frac{d}{R_r} \quad q_2 = 1 + \frac{d}{R_o}$$

$$q_1 = 1 - \frac{(R_r + R_o)}{2 R_r} \quad q_2 = 1 + \frac{(R_r + R_o)}{2 R_o}$$

$$q_1 = 1 - \frac{1}{2} + \frac{R_o}{2 R_r} \quad q_2 = 1 + \frac{1}{2} + \frac{R_r}{2 R_o}$$

$$q_1 = \frac{1}{2} \left( 1 + \frac{R_o}{R_r} \right) \quad q_2 = \frac{1}{2} \left( 3 + \frac{R_r}{R_o} \right)$$

$$q_1 q_2 = \frac{1}{4} \left( 3 + 3 \frac{R_o}{R_r} + \frac{R_r}{R_o} + \frac{R_o R_r}{R_r R_o} \right)$$

$$= \frac{1}{4} \left( 4 + \frac{3 R_o}{R_r} + \frac{R_r}{R_o} \right)$$

$$q_1 + q_2 = \frac{1}{2} \left( 1 + \frac{R_o}{R_r} + 3 + \frac{R_r}{R_o} \right) = \frac{1}{2} \left( 4 + \frac{R_o}{R_r} + \frac{R_r}{R_o} \right)$$

$$\therefore 2 q_1 q_2 = q_1 + q_2$$
3. A 0.1-m-long Nd:YAG laser rod is installed in a 0.4-m-long laser cavity of a Q-switched laser. Determine the approximate value for \( t_s \) and show that it is significantly shorter than \( \tau_u \), the upper laser level lifetime.

\[
\begin{align*}
L & = 0.1 \text{ m (Nd:YAG)} & d & = 0.4 \text{ m} & \eta_c &= \eta_{\text{air}} = 1 \\
\eta_L & = 1.82 & \tau_u & = 230 \mu s \\
9 \tau_u mL & = 12 \\
\text{where } mL \text{ is the effective gain length}
\end{align*}
\]

\[
\therefore m = \frac{12}{9 \tau_u L} = \frac{12}{10 \text{ m}^{-1} \times 0.1 \text{ m}} = 12
\]

The minimum gain duration \( t_s \) for the beam to reach \( I_{\text{sat}} \) is:

\[
t_s = m \left[ \eta_c (d-L) + \eta_L L \right] / c = \frac{12 [1 \cdot (0.4 - 0.1) + 1.82 \cdot 0.1]}{3 \times 10^8}
\]

\[
\leq 1.9 \times 10^{-8} \text{ s} = 19 \text{ ns} \ll 230 \mu \text{s}
\]
4. A 0.1-m–long ruby laser rod has the mirrors coated on the ends of the rod. The output mirror has a reflectivity of 90% and the rear mirror a reflectivity of 99.9%. If there is no excited state absorption (i.e., \( \alpha = 0 \)) in the laser, what would be the fractional loss per pass in the laser cavity, and what would be the decay time of the cavity?

From (8.38)

\[
L_F = \frac{N}{L} - \frac{\ln (R_1 R_2)}{2}
\]

but \( \alpha = 0 \)

\[
\therefore L_F = -\frac{\ln (R_1 R_2)}{\sqrt{2}} = -\frac{1}{2} \ln (R_1 R_2)
\]

\[
= -\frac{1}{2} \ln [(0.999)(0.90)] = +0.053
\]

From (8.39)

\[
T_c = \frac{N \cdot d}{L \cdot L_F} = \frac{1.76 (0.1)}{3 \times 10^8 \cdot 0.053} = 1.1 \times 10^{-8}
\]

\[
= 11 \text{ ns}
\]
5. What would be the maximum Q-switched power output from the ruby laser given in Problem 4 if the mode occupies a diameter of 2 mm within the rod? Assume that the laser gain is 100 times the threshold value. *Hint:* Determine $\Delta N_{ul}$ from the equations of Section 13.10.

\[
\Delta N = \frac{1}{\Delta_{ul} L} \left( \frac{1}{R_1 R_2} \right) \ln \left( \frac{1}{R_1 R_2} \right)
\]

\[
M_0 = 100 \Delta N V = 100 \Delta N \frac{\pi d^2}{4} L
\]

\[
= 100 \frac{\pi d^2}{8} \frac{1}{\Delta_{ul}} \ln \left( \frac{1}{R_1 R_2} \right)
\]

\[
= 100 \frac{\pi (2 \times 10^{-3})^2}{8} \frac{1}{2.5 \times 10^{-24}} \ln \left[ \frac{1}{(0.999)(0.9)} \right]
\]

\[
= \frac{6.68 \times 10^{18}}{2.5 \times 10^{-24}} \ln \left[ \frac{1}{(0.999)(0.9)} \right]
\]

\[
P_{\text{max}} = \frac{M_0 \ h \nu}{2 \ \pi c} = \frac{6.68 \times 10^{18} \ 6.63 \times 10^{-34} \left( \frac{3 \times 10^8}{6.64 \times 10^{-9}} \right)}{2 \ \left( 1.1 \times 10^{-8} \right)}
\]

\[
= 8.7 \times 10^7 \ W
\]
6. Obtain (13.23), the expression for photon density, from (13.22). Obtain the value of $M$ where $\phi$ is a maximum.

\[
\frac{d\phi}{dM} = \left( \frac{M_{m}}{2M} - \frac{1}{2} \right) = \frac{1}{2} \left( \frac{M_{m}}{M} - 1 \right)
\]

\[
d\phi = \left( \frac{M_{m}}{2} \frac{1}{M} - \frac{1}{2} \right) dM
\]

Integrate \( \Rightarrow \) \( \phi = \frac{M_{m}}{2} \ln M - \frac{M}{2} + c \)

\[
= \frac{1}{2} \left( M_{m} \ln M - M \right) + c
\]

when \( \phi = \phi_0 \), \( M = M_0 \)

\( \therefore \) \( \phi_0 = \frac{1}{2} \left( M_{m} \ln M_0 - M_0 \right) + c \)

or \( c = \phi_0 - \frac{1}{2} \left( M_{m} \ln M_0 - M_0 \right) \)

\( \therefore \) \( \phi - \phi_0 = \frac{1}{2} \left[ M_{m} \ln \frac{M}{M_0} - (M - M_0) \right] \)

\( \phi \) is maximum when:

\[
\frac{d\phi}{dM} = \left( \frac{M_{m}}{2} \frac{1}{M} - \frac{1}{2} \right) = 0
\]

\[
\frac{M_{m}}{M} - 1 = 0 \ \text{or} \ M = M_{m}
\]
7. What would be the minimum pulse duration of a mode-locked alexandrite laser? If the separation between mirrors is 2 m and the alexandrite laser rod is 0.1 m long, what would be the separation between mode-locked pulses?

\[ d = 2 \text{ m} \quad L = 0.1 \text{ m} \quad \eta_L = 1.74 \text{ (alexandrite)} \]

\[ \Delta V_{\text{FWHM}} = 2.6 \times 10^{13} \text{ Hz} \quad \eta_c = 1.0 \]

\[ \Delta T_p = \frac{1}{\Delta V_{\text{FWHM}}} = \frac{1}{2.6 \times 10^{13} \text{ Hz}} = 3.85 \times 10^{-14} \text{ s} \]

\[ \Delta T_{\text{sep}} = \frac{1}{\Delta V} = \frac{2}{\eta_c (d - L) + \eta_L L} = \frac{2}{1 (d - L) + \eta_L L} \]

\[ = \frac{2}{(2 \text{ m} - 0.1 \text{ m}) + 1.74 (0.1 \text{ m})} \frac{1}{3 \times 10^8} \]

\[ = 1.38 \times 10^{-8} \text{ s} = 13.8 \text{ ns} \]
8. For a gallium arsenide distributed feedback laser in which only the index of refraction is modulated, what would be the necessary index modulation in order for the laser to operate with this type of resonator? Assume that the laser cavity is 0.5 mm in length. What would be the bandwidth of the laser output?

For index modulation, the threshold is

\[ \eta_1 = \frac{\lambda_0}{L} \frac{\ln G}{\pi G^{1/2}} \]

Let \( G = 1,000 \) from Table 15-18, p. 596.

Then \[ \eta_1 = \frac{\lambda_0}{0.0005} \frac{\ln 1,000}{\pi (1,000)^{1/2}} \]

\[ = \frac{870 \times 10^{-9} \ln 10^3}{5 \times 10^{-4} \pi (1000)^{1/2}} \]

\[ = 3.56 \times 10^{-5} \]

\[ \therefore \Delta \lambda = (3.56 \times 10^{-5}) (870 \times 10^{-9} \text{ m}) \]

\[ = 3.10 \times 10^{-11} \text{ m} = 0.031 \text{ nm} \]
9. A hypothetical gas laser operates at 200 nm in a cavity with mirrors separated by 2 m and is dominated by Doppler broadening with a Doppler width (FWHM) of $4 \times 10^9$ Hz. The gain for a single pass through the amplifier is 12%, and each mirror has a reflectivity of 98%, a transmission of 1%, and a scattering loss of 1%. There is also a scattering loss of 0.5% at each Brewster window. If this laser were mode-locked, what would be the maximum number of modes that could contribute to producing mode-locked pulses, and what would be the minimum pulse width of each pulse? What would be the separation between pulses?

![Diagram of laser cavity with gain and loss curves]

\[ \Delta v_D = 4 \times 10^9 \text{ Hz} \]

\[ R = 0.98 \]

\[ T = 0.01 \]

\[ \alpha = 0.01 + 2(0.005) = 0.02 \]

Total cavity losses 3%/pass

\[ q_D(v) = \frac{\sqrt{\ln 2}}{16 \pi^3} \frac{c^2 A_{ul}}{N^2 2 \pi \Delta v_D} \left[ N_0 - \frac{N_0}{N_0} \right] e^{-\left[ \frac{4 \ln 2 (2 - \alpha v)}{\Delta v_D} \right]} \]

at \( v = v_D \)

\[ q_D(v) = \frac{\sqrt{\ln 2}}{16 \pi^3} \frac{c^2 A_{ul}}{2 \pi \Delta v_D} \]

or \( \exp = 1 \) when gain is 12%

let \( v - v_D = \Delta v \) and solve when \( \exp = \frac{1}{4} \) or 3%

\[ \exp = -\frac{4 \ln 2 \Delta v^2}{\Delta v_D^2} = \frac{1}{4} \]

\[ \exp = \frac{4 \ln 2 \Delta v^2}{\Delta v_D^2} = 4 \]

\[ 4 \ln 2 \left( \frac{\Delta v}{\Delta v_D} \right)^2 = 1.386 \Rightarrow \left( \frac{\Delta v}{\Delta v_D} \right)^2 = \frac{1.386}{4 \ln 2} \]

\[ \Delta v = \sqrt{0.50} \Delta v_D = 0.707 \Delta v_D \]

\[ = 0.707 (4 \times 10^9 \text{ Hz}) = 2.82 \times 10^7 \text{ Hz} \]

\[ \Delta v_{sep} = \frac{\Delta v}{2d} = \frac{3 \times 10^8}{2 \text{ cm}} = 7.5 \times 10^7 \text{ Hz} \]

\# modes = \[ \frac{2.82 \times 10^9}{7.5 \times 10^7} + 1 = 37.6 + 1 = 38 \text{ modes} \]

\[ \Delta t_{sep} = \frac{2d}{c} = 13.33 \text{ ns} \]

\[ \Delta t_p = \frac{1}{7.5 \times 10^7 \text{ Hz}} = 3.5 \times 10^{-15} \]
10. You have the opportunity to observe a three-mirror laser cavity (such as that shown in Figure 13-31) in which the laser is producing mode-locked pulses at intervals of 12.0 ns. The focal length of both $M_1$ and $M_2$ is 0.1 m. What is the distance between $M_2$ and $M_3$? If the mode-locked pulses have a width of 133 fs and you know that the laser is mode-locked over the FWHM of the laser gain medium, what would you suspect is the gain medium?

\[
\text{for } M_1 + M_2, \quad f = 0.1 \text{m}
\]
\[
\Delta t_{\text{sep}} = 12 \text{ns} = \frac{2d}{c} \Rightarrow d = 1.80 \text{m}
\]

distance of $M_2$ to gain cell is 0.1 m

gain cell to $M_1$ is $2f = R = 0.2$ m

\[
\Delta \nu_{\text{sep}} = \frac{c}{2d} = \frac{3 \times 10^8}{2 \times (1.8)} = \frac{333 \times 10^7}{2} \text{Hz}
\]

\[
\Delta \tau_0 = \frac{1}{\Delta \nu_{\text{sep}} N} = \frac{133 \times 10^{-15}}{\text{sec}} = \frac{1}{\text{gain bandwidth}}
\]

\[\text{gain bandwidth} = 7.52 \times 10^{12} \text{Hz}\]

probably Nd:Glass with bandwidth of $7.5 \times 10^{12} \text{Hz}$